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On the Parameterization of the Autoconversion Process. Part I: Analytical Formulation of the Kessler-Type Parameterizations

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Abstract

Various commonly used Kessler-type parameterizations of the autoconversion of cloud droplets to embryonic raindrops are theoretically derived from the same formalism by applying the generalized mean value theorem for integrals to the general collection equation. The new formalism clearly reveals the approximations that are implicitly assumed in these different parameterizations. It is shown that the different parameterizations can be generalized into a common expression, and that their differences lie in the characterization of the effect of the spectral dispersion of the cloud droplet size distribution on the autoconversion rate. A new Kessler-type parameterization is derived from the formalism. This new parameterization eliminates the incorrect and/or unnecessary assumptions inherent in the existing ones, exhibits a different dependence on liquid water content and droplet concentration, and provides theoretical explanations for the multitude of values assigned to the tunable coefficients associated with the commonly used parameterizations. Relative dispersion of the cloud droplet size distribution is explicitly included in the new parameterization, allowing for investigation of the influences of the spectral dispersion on the autoconversion rate, and hence on the second indirect aerosol effect.

Key words: autoconversion rate, generalized mean value theorem for integration, cloud liquid water content, droplet concentration, relative dispersion.

1. Introduction

Rain is initiated in liquid water clouds by collision and coalescence of cloud droplets wherein larger droplets with higher settling velocities collect smaller droplets and become embryonic raindrops. This so-called autoconversion process is usually the dominant process that leads to the formation of drizzle in stratiform clouds. Accurate parameterization of the autoconversion process in atmospheric models of various scales (from cloud-resolving models to global climate model) is important for understanding the interactions between cloud microphysics and cloud dynamics (Chen and Cotton 1987), for the forecasting of the freezing drizzle formation and aircraft icing (Rasmussen et al. 2002), and for improving the treatment of clouds in climate models (Rotstayn 2000).

Kessler (1969) proposed a simple parameterization that linearly relates the autoconversion rate to the cloud liquid water content, and this parameterization has been widely used in cloud-related modeling studies because of its simplicity. But this simple parameterization leaves much to be desired, as it is well known that the autoconversion rate is a function of not only of the liquid water content, but also the cloud droplet number concentration and the spectral dispersion of the cloud droplet size distribution. Over the last several decades, much effort has been devoted to improving the original Kessler parameterization by including the effect of the droplet concentration as well as liquid water content (Manton and Cotton 1977; Tripion and Cotton 1980; Liou and Ou 1989; Baker 1993). The effort to improve parameterization of the autoconversion rate has been recently reinforced by an increasing interest in cloud-climate interactions, and particularly in studies of the second indirect aerosol effect (Boucher et al. 1995; Lohmann and Fleichter 1997; Rotstayn 2000).

Without loss of generality, all of the Kessler-type parameterizations can be written as

$$P = cLH\left(y - y_c\right), \tag{1}$$

where P is the autoconversion rate in g cm^{-3} s^{-1}, c is an empirical coefficient in unit of s^{-1} (hereafter conversion coefficient), and L is the cloud liquid water content in g cm^{-3}. The Heaviside step function $H(y-y_c)$ is introduced to describe a threshold y_c (hereafter threshold coefficient) below which the autoconversion is negligibly small. The meaning of y is different in different parameterizations; for example, y represents the cloud liquid water content in the original Kessler parameterization, whereas it represents the mean volume radius in the Manton and Cotton expression, and the mean radius of the fourth moment in the parameterizations of Liou and Ou (1989), Baker (1993) and Boucher et al (1995). It is noteworthy that while the autoconversion rate is also formulated in terms of the cloud water mixing ratio instead of the liquid water content, transformation between these two equivalent formulations is straightforward.

A common problem with the Kessler-type parameterizations is that they collectively lack a solid theoretical foundation, approximations associated with their use are not clear, and the logical connections between the various Kessler-type parameterizations are not well understood. Here we first derive the various existing Kessler-type parameterizations by applying the generalized mean value theorem to the general collection equation. This derivation readily reveals the distinctions between, and approximations of these different parameterizations. The existing Kessler-type parameterizations are then generalized into a unified expression that includes the effect of the spectral dispersion of the cloud droplet size distribution as well as the droplet concentration and liquid water content. A new Kessler-type parameterization that eliminates the incorrect and/or unnecessary assumptions inherent in the existing parameterizations is further developed, and applied to explain the multitude of the empirical coefficients associated with the existing Kessler-type parameterizations. The effect of the spectral dispersion on the autoconversion rate is also discussed.

2. Reexamination of Typical Kessler-Type Parameterizations

As discussed above, one of the problems shared by the existing Kessler-type parameterizations is the lack of a physical basis for their formulation. The purpose of this section is to show that application of the generalized mean value theorem for integrals to the general equation for the autoconversion rate can provide the required theory.

a. Autoconversion rate and Generalized mean value theorem for integrals

We first recapitulate the expression for the autoconversion rate and the generalized mean value theorem for integrals that will be used in this work. From the continuous collection equation, the mass growth rate of a collector drop of radius R falling through a population of smaller droplets having a cloud droplet size distribution n(r) is given by (Pruppacher and Klett 1997)

$$\frac{dm(R)}{dt} = \int K(R,r)m(r)n(r)dr \qquad (2)$$

The autoconversion rate P is obtained by further integrating (2) over all collector drops:

$$P = \int \frac{dm}{dt} n(R)\,dR = \int n(R)\,dR \int K(R,r)m(r)n(r)dr \qquad (3)$$

The interval of the integration is from the smallest cloud droplets to the smallest raindrop, and is omitted throughout the paper for simplicity. It is well known from standard calculus textbooks (e.g., Spiegel 1992) that, if f(x) and g(x) are continuous in the interval $x \in [a, b]$, and g(x) does not change sign in this interval, then there is a point $x_\xi \in (a, b)$ such that

$$\int_a^b f(x)g(x)\,dx = f(x_x)\int_a^b g(x)\,dx\,. \tag{4}$$

It will be shown below that the application of the generalized mean value theorem for integrals to (3) provides a unified basis for the various Kessler-type parameterizations.

b. Derivation of the typical Kessler-type parameterization

Kessler (1969) intuitively proposed an expression for the autoconversion rate such that

$$P_K = a_K\left(L - L_c\right), \tag{5a}$$

where L_c is the threshold liquid water content below which the autoconversion rate is assumed to be so small that the empirical coefficient $a_k = 0$ when $L \leqslant L_c$, and $a_k > 0$ when $L > L_c$. This parameterization can also be expressed through the Heaviside step function, i.e.,

$$P_K = c_K LH\left(L - L_c\right) \tag{5b}$$

Comparison of (5b) with (5a) yields

$$c_K = a_K\left(1 - \frac{L_c}{L}\right), \tag{5c}$$

Equation (5c) provides an explanation for the results obtained by Kessler (1969) that increasing a_k affects precipitation development in much the same way as does increasing the conversion threshold, because the autoconversion rate increases when a_k increases or the threshold liquid water content decreases for a given liquid water content.

The Kessler parameterization can also be derived by applying the generalized mean value theorem for integrals to (3) as follows. Application of the generalized mean value theorem to the first integral of (3) yields

$$P = \int K(R, r_K) n(R)\, dR \int m(r) n(r) dr = L \int K(R, r_K) n(R)\, dR \qquad (6)$$

where r_K is between the smallest cloud droplet and the smallest raindrop. Further application of the generalized mean value theorem to (6) yields

$$P = K(R_K, r_K) NL = c_K L, \qquad (7)$$

where R_K is between the smallest droplet and the smallest raindrop, N is the total number concentration of cloud droplets, and $K(R_K, r_K)$ represents the "average" collection kernel. Equation (7) becomes the Kessler parameterization if the conversion rate satisfies (5c). The above derivation clearly shows that the original Kessler parameterization with constant values of a_k and L_c results from the assumption of an "average", fixed collection kernel that is independent of droplet radius and proportional to the quantity given by $(1-L_c/L)N^{-1}$. These assumptions are clearly not valid.

Manton and Cotton (1977, see also Tripoli and Cotton 1980) formulated a similar expression for the autoconversion rate,

$$P_{MC} = c_{MC} LH(L - L_c). \qquad (8)$$

Unlike the original Kessler parameterization, however, the conversion coefficient was further expressed as

$$c_{MC} = p\, E_{MC} R_3^2 V\left(R_3 \right) N\,, \tag{9}$$

where E_{MC} represents an average collection efficiency associated with the autoconversion process, R_3 is the mean volume radius, and V is the terminal velocity of a droplet of radius R_3. They also argued that the threshold of the autoconversion process was determined by the mean volume radius instead of by the liquid water content such that

$$L_c = \frac{4p r_w}{3} R_{3c}^3 N\,, \tag{10}$$

where R_{3c} is the threshold mean volume radius, and ρ_w is the density of water. Manton and Cotton used E_{MC} = 0.55, and R_{3c} = 10 μm.

The Manton and Cotton expression can also be derived by applying the mean value theorem to the collection equation, but in a slightly different way than for the original Kessler parameterization. The collection kernel K (R, r) depends generally on the collection efficiency E and the terminal velocity V, and is given by

$$K\left(R,r\right) = E\left(R,r\right) p\, \left(R+r\right)^2 \left[V\left(R\right) - V\left(r\right)\right] \tag{11}$$

Because cloud droplets are so small, this equation can be simplified by assuming that

$$\left(R+r\right)^2 \approx R^2\,, \tag{12a}$$

and $$V\left(R\right) - V\left(r\right) \approx V\left(R\right) \tag{12b}$$

Substitution of (12a), (12b) and (11) into (3) yields

$$P = p \int R^2 V\left(R\right) n\left(R\right) dR \int E\left(R,r\right) m\left(r\right) n\left(r\right) dr \tag{13}$$

Application of the generalized mean-value theorem to the first integral of (13) yields

$$P = p\, L \int R^2 V\left(R\right) n\left(R\right) E\left(R, r_{MC}\right) dR \tag{14}$$

Further application of the generalized mean-value theorem to (14) yields

$$P = p\,LR_{MC}^{2}V\left(R_{MC}\right)E_{MC}\int n\left(R\right)dR = p\,E_{MC}R_{MC}^{2}V\left(R_{MC}\right)NL \tag{15}$$

Comparison of (15) to (8) and (9) shows that (15) reduces to the Manton and Cotton parameterization under the assumption of $R_{MC} = R_3$. This assumption is invalid except in the case of a monodisperse cloud droplet size distribution.

The familiar form of the Manton and Cotton parameterization can be derived by assuming that the terminal velocity of the drop R is well described by the Stokes law

$$V\left(R\right) = k_1 R^2, \tag{16}$$

where $\kappa_1 = 1.19 \times 10^6\ cm^{-1}s^{-1}$ is the Stokes constant. Substitution of (16) into (15) yields

$$P_{MC} = p k_1 E_{MC} R_3^4 NL \tag{17a}$$

Substitution into (17a) of the expression relating the mean volume radius to the liquid water content and droplet concentration yields the familiar form of the Manton and Cotton parameterization,

$$P_{MC} = a_{MC} N^{-1/3} L^{7/3} H(R_3 - R_{3c}), \tag{17b}$$

where the parameter $a_{MC} = p k_1 \left(\dfrac{3}{4 p r_w}\right)^{4/3} E_{MC}$ (17c)

The Heaviside function $H(R_3 - R_{3c})$ is introduced to consider the threshold process such that the autoconversion rate is negligibly small when $R_3 < R_{3c}$.

A major improvement of the Manton and Cotton parameterization over the original Kessler parameterization is inclusion of the droplet concentration as a dependent variable in formulation of the autoconversion rate, which enables one to differentiate between air mass types. Another improvement is that the threshold is determined by the volume mean radius rather than the liquid water content; this change makes physical sense because a cloud with a large liquid water content, a large number of droplets and therefore a small mean volume radius will not rain. It is evident from the above derivation that these improvements result from relaxing the assumption of a fixed the collection kernel (independent of the droplet radius) inherent in the original Kessler parameterization. The derivation also exposes the following deficiencies remaining in the Manton and Cotton parameterization: fixed collection efficiency, terminal velocity and $R_{MC} = R_3$.

Several parameterizations that are slightly different from the Manton and Cotton parameterization have been subsequently proposed. Instead of applying the mean-value theorem to the integral of (14) before substituting the Stokes law for the terminal velocity, Liou and Ou (1989) relaxed the assumption of fixed terminal velocity by first applying the Stokes law for the terminal velocity, and obtained the autoconversion rate

$$P = pk_1 L \int E\left(R, r_{MC}\right) R^4 n(R)\, dR \tag{18}$$

A subsequent application of the generalized mean-value theorem to (18) yields

$$P_{LO} = pk_1 E_4 L \int R^4 n(R)\, dR = pk_1 E_4 R_4^4 NL \tag{19}$$

where E4 is the average collection efficiency associated with (18), and R_4 is the mean radius of the fourth moment defined as

$$R_4 = \left(\frac{\int R^4 n(R)\, dR}{N} \right)^{1/4} \tag{20}$$

They assumed a fixed linear relation between R_4 and the mean square radius R_2, $R_4 = 1.247\ R_2$, and investigated sensitivities of cloud radiative properties to the mean square radius.

In investigation of the behavior of cloud condensation nuclei in the marine cloud-topped boundary layer, Baker (1993) used a similar parameterization, but assumed R_4 is equal to the mean volume radius such that

$$P_{Baker} = pk_1 \left(\frac{3}{4pr_w} \right)^{4/3} E_4 g\, N^{-1/3} L^{7/3} H(R_3 - R_{3c}), \qquad (21)$$

where E_{LO} = 0.55, R_{3c} = 10 μm, and the empirical multiplier γ, which varies from 0.01 and 0.1, was introduced to make the autoconversion rate smaller. In their GCM study, Boucher et al. (1995) assumed a fixed linear relation between R_4 and the mean volume radius, $R_4 = 1.1R_3$, and obtained a autoconversion parameterization given by

$$P_{Boucher} = a_B N^{-1/3} L^{7/3} H\left(R_4 - R_{4c}\right), \qquad (22a)$$

$$a_B = pk_1 \left(\frac{3}{4pr_w} \right)^{4/3} (1.1)^4 E_4 g \qquad (22b).$$

They also studied the sensitivity to the value of the threshold radius. Note that unlike Baker (1993), they found that the a value of γ = 1 generated more reasonable results. A value of E_4 = 0.55 was also used in this study.

3. New Parameterizations

a. Generalized R_4 parameterization

Compared to the Manton-Cotton parameterization, one of the features shared by the Baker and the Boucher parameterizations is that the mean volume radius R_3 in the Manton-Cotton parameterization is replaced by the mean radius of the fourth moment R_4 in both the conversion and the threshold coefficients. Values of the two average collection efficiencies, E_{MC} and E_4, may differ to some degree. These differences arise because the Baker and Boucher parameterizations eliminate the assumption of fixed terminal velocity. Furthermore, as will become evident later, the linear relation between R_4 and R_3 assumed in the Baker and Boucher parameterizations is easier to physically justify than the assumption of $R_{MC} = R_3$ in the Manton and Cotton parameterization. However, the differences between the three parameterizations are minimal in practice, because the α parameters (α_{MC}, α_{Baker}, and $\alpha_{Boucher}$), and the threshold radii are arbitrarily tuned in most modeling studies. For this reason, the three parameterizations will hereafter be lumped together, and referred to as the traditional R_4 parameterizations to emphasize the important role of the fourth moment [see Eq. (19)].

The Baker and Boucher parameterizations can be generalized into a common expression by assuming a general linear relation between the mean volume radius and the mean radius of the fourth moment such that

$$R_4 = b_4 R_3 , \tag{23}$$

where β_4 is a nondimentional parameter depending on the spectral shape of the cloud droplet size distribution. Application of this expression gives the generalized R_4 parameterization

$$P_4 = a_4 N^{-1/3} L^{7/3} , \tag{24a}$$

$$a_4 = p k_1 \left(\frac{3}{4 p r_w} \right)^{4/3} E_4 b_4^4 . \tag{24b}$$

The differences between the three traditional R_4 parameterizations become evident from the above equations. The Baker parameterization is a special case of the generalized R_4 parameterization with $\beta_4 = 1$. In practice, the Manton and Cotton parameterization can also be considered a special case with $\beta_4 = 1$. As will be shown below, β_4 is an increasing function of the relative dispersion of the cloud droplet size distribution, a common measure of the spectral shape defined as the ratio of the standard deviation to the mean radius of the cloud droplet size distribution. A value of $\beta_4 = 1$ is equivalent to assuming a monodisperse cloud droplet size distribution. The Boucher parameterization corresponds to a special case of $\beta_4 = 1.1$, suggesting the assumption of a larger, yet fixed spectral dispersion for the cloud droplet size distribution. Therefore, the primary differences between the traditional R_4 parameterizations reflect their different choices for the relative dispersion of the cloud droplet size distribution. Obviously, the assumption of fixed spectral dispersion, monodisperse or not, is troublesome. In fact, the effect of the spectral dispersion can be explicitly investigated if we assume that the cloud droplet size distribution can be well described the gamma distribution. Under this condition, β_4 is easily shown to be uniquely related to the spectral dispersion of the cloud droplet size distribution by,

$$b_4 = \frac{\left(1+3e^2\right)^{1/4}}{\left[\left(1+2e^2\right)\left(1+e^2\right)\right]^{1/12}} , \qquad (25)$$

where ε represents the relative dispersion.

b. A New R_6 parameterization

Although the various R_4 parameterizations are significant improvements of the original Kessler parameterization, they still suffer from the implicit deficiency that the collection efficiency is treated as a constant. This assumption is obviously incorrect because it means collections between droplets of nearly the same size are just as numerous as those between droplets of very different sizes. Baker (1993) discussed this deficiency, and introduced a multiplicative parameter γ ranging from 0.01 to 0.1 to adjust for this effect. Here we develop a new parameterization that does not assume a fixed collection efficiency.

Long (1974) showed that the collection kernel can be well approximated by

$$K\left(R,r\right)=k_2 R^6 \,, \tag{26}$$

where the coefficient $k_2 \approx 1.9\times10^{11}$ in cm^{-3}s^{-1}, R is in cm, and the collection kernel K is in cm^{3} s^{-1}. Substitution of (26) into (3) yields

$$P_6 = k_2 L\int R^6 n\left(R\right)dR = k_2 N R_6^6 L \,, \tag{27}$$

where N is in cm^{-3}, R_6 is the mean radius of the sixth moment in cm, L is in g cm^{-3}, and P_6 is g cm^{-3}s^{-1}. Similar to the generalized R_4 parameterization, we assume a general linear relation between the mean volume radius and the mean radius of the sixth moment, $R_6 = b_6 R_3$. This relation leads to the following expressions

$$P_6 = hN^{-1}L^3 H\left(R_6 - R_{6c}\right), \tag{28a}$$

$$h=\left(\frac{3}{4\pi\rho_w}\right)^2 k_2 b_6^6 \tag{28b}$$

It is clear from the above equations that the autoconversion rate of this new parameterization exhibits a different dependence on the liquid water content and droplet concentration than the R_4 parameterizations. It also suggests that the threshold is determined by the mean radius of the sixth moment rather than the mean radius of the fourth moment. The new parameterization also exhibits different dependence on the relative dispersion than the generalized R_4 parameterization. For the purpose of comparison, the new R_6 parameterization can also be rewritten in the forms of the original Kessler and the R_4 parameterizations such that

$$P_6 = c_6 L H\left(R_6 - R_{6c}\right) = \alpha_6 N^{-1/3} L^{7/3} H\left(R_6 - R_{6c}\right), \qquad (29a)$$

$$\alpha_6 = \left(\frac{3}{4\pi\rho_w}\right)^2 k_2 \beta_6^6 \left(\frac{L}{N}\right)^{2/3}, \qquad (29b)$$

$$c_6 = \alpha_6 N^{-1/3} L^{4/3}. \qquad (29c)$$

Again, under the assumption that the cloud droplet size distribution can be well described the gamma distribution, the relationship between β_6 and the relative dispersion (ε) is easily shown to be

$$\beta_6 = \left[\frac{\left(1+3\varepsilon^2\right)\left(1+4\varepsilon^2\right)\left(1+5\varepsilon^2\right)}{\left(1+\varepsilon^2\right)\left(1+2\varepsilon^2\right)}\right]^{1/6}. \qquad (30)$$

The above equations suggests a different dependence of the conversion coefficient than the R_4 parameterizations. Furthermore, the new R_6 parameterization indicates that the α coefficient in the R_4 parameterizations should also be a function of the liquid water content, droplet concentration and relative dispersion instead of a constant as assumed in the traditional R_4 parameterizations. This dependence provides a plausible explanation for the wide range of α (or γ) values that have been assigned by investigators in use of the traditional R_4 parameterizations.

4. Discussion

To facilitate comparison, all the Kessler-type parameterizations discussed above are summarized in Table 1 in the forms of the Kessler parameterization and the R_4 parameterization because of their widespread use. Also given in the table are the major approximations and assumptions associated with these parameterizations as revealed by the common derivation process. It is clear from this table that the conversion coefficients in all of the R_4 parameterizations exhibit the same dependence on cloud liquid water content and droplet concentration; the dependence of the new R_6 parameterization on the liquid water content and droplet concentration is considerably different than for the R_4 parameterization. Both the new R_4 and R_6 parameterizations also have a conversion coefficient that depends on the relative dispersion as well, although the details of this dependency are quantitatively different in the two parameterizations.

Examination of our new R_6 parameterization also provides an explanation for a number of long-standing issues associated with the original Kessler parameterization as well as the various R_4 parameterizations. For example, such a wide range of values have been assigned to the coefficient a_k in studies using the original Kessler parameterization that in practice, it has been often considered to be arbitrarily tunable (e.g., Kessler 1969; Liu and Orville 1969; Ghosh et al. 2000). It is evident from the new R_6 parameterization that the wide range of values assumed for a_k may stem from the variabilities in the liquid water content, droplet concentration and relative dispersion that are not properly accounted for in the original Kessler parameterization. Similar to the arbitrary tunability of the coefficient a in the original Kessler parameterization, a wide range of values have been also assigned to the α coefficient in modeling studies using the traditional R_4 parameterizations (Baker 1993; Boucher et al. 1995). For example, the range of γ from 0.01 to 1 as suggested by Baker (1993) and Boucher et al. (1995) alone leads to a difference of three orders of magnitude in α. The new R_6 parameterization again shows that the multitude of values that have been assigned to α may be due to the combined variabilities in liquid water content, droplet concentration and relative dispersion.

Furthermore, both the generalized R_4 parameterization and the new R_6 parameterization explicitly account for the effect of the spectral shape through the dependency of β_4 and β_6 on the relative dispersion [Eq. (25) for β_4 and Eq. (30) for β_6]. This is a desirable feature because the effect of spectral shape on the autoconversion rate is not well understood and quantified despite the fact that it is well known that the relative dispersion has an important role in the autoconversion process. In particular, none of the previous Kessler-type parameterizations includes the relative dispersion as a dependent variable.

However, the dependency of the autoconversion rate on the relative dispersion are quantitatively different for the new R_4 and R_6 parameterizations. Figure 1 shows β_4^4 and β_6^6 as a function of the relative dispersion. Two points can be drawn from this figure. First, as expected, both β_4 and β_6 (therefore the autoconversion rate) increases with increasing broadness of the cloud droplet size distribution. Second, compared to our new R_6 parameterization, the generalized R_4 parameterization underestimates the effect of spectral dispersion on the autoconversion rate by up to an order of magnitude.

The relative dispersion of the cloud droplet size distribution also influences on the autoconversion rate by affecting the threshold radius. This effect is illustrated in Fig. 2, which shows β_4 and β_6 as a function of the relative dispersion. Since the threshold radius for the generalized R_4 parameterization and the new R_6 parameterization are respectively $R_4 = \beta_4 R_3$ and $R_6 = \beta_4 R_3$, both parameterizations tend to have threshold radii larger than the threshold mean volume radius used in the Manton and Cotton parameterization and the Baker parameterization. For the generalized R_4 parameterization, the underestimation by the mean volume radius can reach a factor of 1.4. For the new R_6 parameterization, the underestimation can be larger than a factor of 2. The difference between the generalized R_4 parameterization and the new R_6 parameterization can also reach a factor of 1.5.

These results suggest that the effect of the spectral dispersion alone could cause an uncertainty of up to a factor 2 in the threshold radius. According to few limited studies (Boucher et al. 1995), differences of this magnitude are large enough to significantly affect the results of GCM simulations. We have recently shown that the relative dispersion significantly affects cloud radiative properties (Liu and Daum 2000) as well as the evaluation of the Twomey effect (Liu and Daum 2002; Rotstayn et al. 2003). These dispersion-dependent parameterizations can also be coupled with the corresponding relationship between the relative dispersion and pre-cloud aerosol properties (Liu and Daum 2002) to address the influence of the relative dispersion on the second indirect aerosol effect.

5. Concluding Remarks

The typical autoconversion parameterizations of the Kessler type that have been widely used in cloud-related modeling studies are theoretically derived and analyzed by applying the generalized mean value theorem for integrals to the general collection equation. The approximations implicitly assumed in these parameterizations, their logical connections and the improvements are revealed by the derivations. It is shown that the original Kessler parameterization implicitly assumes a fixed collection kernel. The Manton and Cotton parameterization improves the original Kessler parameterization by relaxing the assumptions of a fixed collection kernel to a fixed collection efficiency and fixed terminal velocity. The Baker and Boucher parameterizations physically improve the Manton and Cotton parameterization by eliminating the assumption of a fixed terminal velocity. It is also demonstrated that the Manton and Cotton parameterization, the Baker parameterization and the Boucher parameterization can actually be considered special cases of a generalized R_4 parameterization that assume different, yet fixed values of the relative dispersion.

A new R_6 parameterization is derived by further eliminating the assumption of a fixed collection efficiency inherent in the various R_4 parameterizations. The new parameterization relates the autoconversion rate to the sixth moment of the cloud droplet size distribution, and represents the physics of the autoconversion process better than those that have been commonly used. Furthermore, the new parameterization indicates that the wide range of values chosen for both the a coefficient in the original Kessler parameterization and the α coefficient in the traditional R_4 parameterizations are in fact mainly due to the variabilities in cloud liquid water content, droplet concentration and relative dispersion in ambient clouds. The practice of arbitrarily tuning coefficients (a and α in the original Kessler parameterization and the R_4 parameterizations) to match some constraints in modeling studies is therefore misleading; critical information is lost in the tuning process.

In comparison with the commonly used parameterizations (traditional R_4 parameterizations and the original Kessler-type parameterization), the generalized R_4 parameterization and the new R_6 parameterization have an additional advantage because they can be used to study the effect of the relative dispersion on the autoconversion rate. Our preliminary analysis indicates that the effect of the relative dispersion is too large to be ignored in the parameterization of the autoconversion process.

It is noteworthy that several parameterizations have been also developed by curve-fitting the autoconversion rate calculated from detailed microphysical models (Berry 1968; Behgeng 1994; Kogan 2000). They are in general agreement with the new

R_6 parameterization, suggesting a stronger dependence of the autoconversion rate on the liquid water content and droplet concentration than that given by either the original Kessler parameterization or the various R_4 parameterizations. Detailed comparison of the new R_6 parameterization with these model-based parameterizations will be presented in Part II of this series.

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Figure Caption

Figure 1. Dependence of $\beta_6{}^6$ (solid line) and $\beta_4{}^4$ (dashed line) on the relative dispersion.

Figure 2. Dependence of β_6 (solid line) and β_4 (dashed line) on the relative dispersion.

Table 1. Summary of the Kessler-Type Autoconversion Parameterizations

$$P = cLH(y - y_c) = a\, N^{-1/3} L^{7/3} H(y - y_c)$$

Parameterizations	Assumptions	Conversion Coefficient c	Threshold y_c
Kessler	fixed collection kernel	$c_K = a_K \left(1 - \frac{L_c}{L}\right)$	L_c
Manton & Cotton	Fixed collection efficiency, monodisperse spectrum & fixed terminal velocity	$c_{MC} = a_{MC} N^{-1/3} L^{4/3}$ $a_{MC} = pk_1 \left(\frac{3}{4pr_w}\right)^{4/3} E_{MC}$	R_{3c}
Baker	Fixed collection efficiency & monodisperse spectrum	$c_{Baker} = a_{Baker} N^{-1/3} L^{4/3}$ $a_{Baker} = pk_1 \left(\frac{3}{4pr_w}\right)^{4/3} E_4$	R_{3c}
Boucher	Fixed collection efficiency & fixed, broader spectrum	$c_{Boucher} = a_{Boucher} N^{-1/3} L^{4/3}$ $a_{Boucher} = pk_1 \left(\frac{3}{4pr_w}\right)^{4/3} E_4 (1.1)^4$	R_{4c}
Generalized R_4	Fixed collection efficiency	$c_4 = a_4 N^{-1/3} L^{4/3}$ $a_4 = pk_1 \left(\frac{3}{4pr_w}\right)^{4/3} E_4 b_4^4$	R_{4c}
New R_6	None of above	$c_6 = a_6 N^{-1/3} L^{4/3}$ $a_6 = k_2 \left(\frac{3}{4pr_w}\right)^2 b_4^4 \left(\frac{L}{N}\right)^{2/3}$	R_{6c}

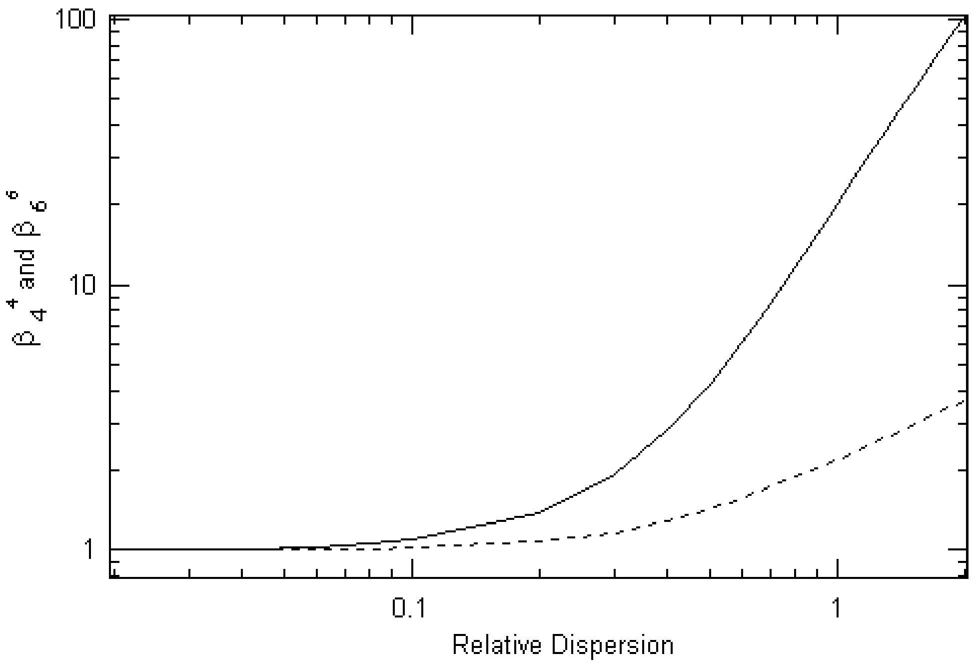

Fig. 1.

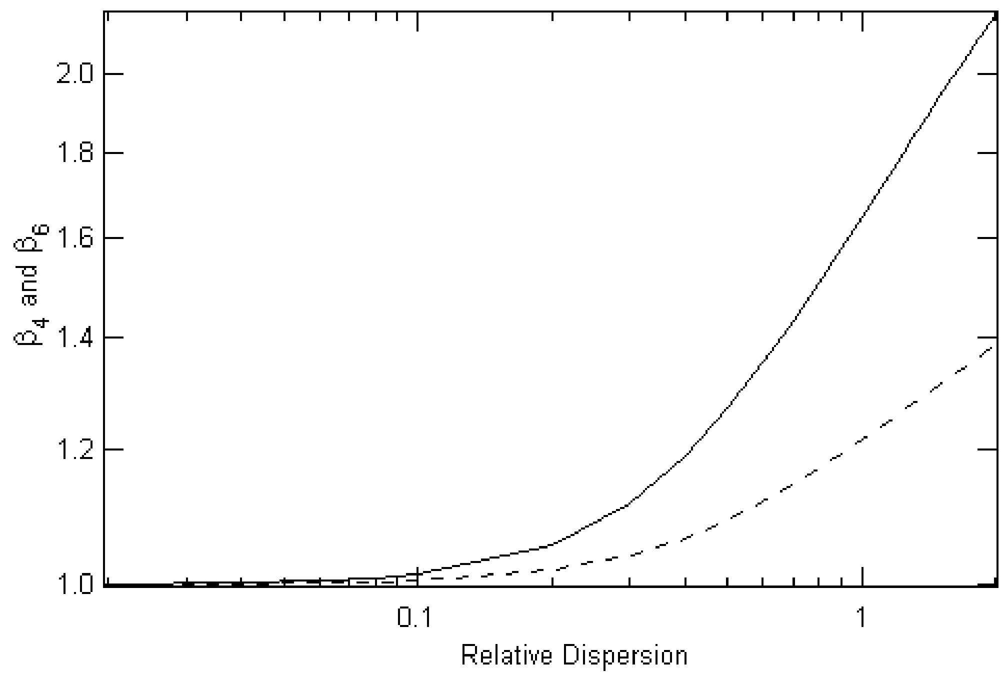

Fig. 2